Journal of Engineering Physics and Thermophysics, Vol. 74, No. 6, 2001

LONGITUDINAL OSCILLATIONS OF AN ORBITAL CABLE SYSTEM UNDER THE THERMAL ACTION ON THE CABLE

V. V. Korovin and A. N. Sdobnikov

UDC 629.7.087.22

Longitudinal oscillations of an orbital cable system (OCS) composed of two end masses and a connecting cable are considered. The change in the temperature of the cable in orbital motion is responsible for the disturbing action. It is evident from the analysis of the natural frequencies of the OCS longitudinal oscillations that the fundamental tone can coincide with one of the disturbance harmonics. In the conservative dynamic model of an OCS, the complete attenuation of the cable is attained at resonance in several circuits of the orbit.

Calculation of a construction for the thermal action of temperature is usually performed in a static statement, i.e., without account for inertial forces. Orbital cable systems are extended and nonrigid objects which are characterized by low natural frequencies of oscillations. A connecting cable is a thermally thin body whose temperature under orbital conditions changes rapidly on transition from the illuminated portion of the orbit to a shadow one, and conversely (intersection of the terminator line). The indicated features of an orbital cable system (OCS) require consideration of the temperature deformation of the cable in a dynamic statement, i.e., with account for inertial forces acting on the cable and end bodies.

We will consider a two-mass OCS (Fig. 1). The change in the cable temperature by ΔT causes deformation $\varepsilon_{th} = \alpha\Delta T$. The cross sections of the cable gain longitudinal displacements under the thermal deformation $u_{th}(x, t) = \varepsilon_{th}(t)x$, where x is the cross-section coordinate, reckoned along the x axis from point C, which is the center of mass. Under temperature deformation, the center of mass is not displaced. A dynamic-equilibrium equation for the cable element of length dx is as follows:

$$m\ddot{u}dx - \mathrm{EF}\,(u - u_{th})''\,dx = 0\,, \tag{1}$$

in which u is the total displacement of the cable element and u_{th} is the displacement caused by the thermal deformation. Equation (1) reflects the longitudinal oscillations of the rod. The cable does not work in compression, but in the OCS it is constantly extended by a static tensile force N_{st}. If the amplitude of the longitudinal force in oscillations N_d is smaller than the amplitude of the static force, then the behavior of the cable is described by Eq. (1). The relation between u and u_{th} will be established in the form

$$u = u_{th} + u_d\,, \tag{2}$$

where u_d is the elastic (dynamic) displacement. Then

$$m\ddot{u}_d dx - \mathrm{EF}u_d''\,dx = -\,m\ddot{u}_{th}dx$$

or

N. É. Bauman Moscow State Technical University, Moscow, Russia; email: departm1@sm.bmstu.ru.
Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 74, No. 6, pp. 73–77, November–December, 2001. Original article submitted February 12, 2001.

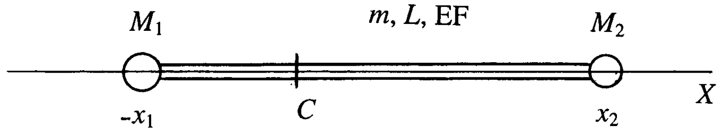

Fig. 1. Calculational scheme of the OCS in the case of longitudinal oscillations.

$$\ddot{u}_{\rm d} - a^2 u''_{\rm d} = -\ddot{u}_{\rm th}, \tag{3}$$

where $\ddot{u}_{\rm th}(x, t) = \alpha \ddot{T}(t) x$ and $a^2 = {\rm EF}/m$.

In conformity with Fig. 1, the boundary conditions are as follows:

$$\begin{aligned} &\text{for } x = -x_1 \quad M_1 \ddot{u}_{\rm d} - {\rm EF} u'_{\rm d} = -M_1 \alpha \ddot{T} x; \\ &\text{for } x = x_2 \quad M_2 \ddot{u}_{\rm d} + {\rm EF} u'_{\rm d} = -M_2 \alpha \ddot{T} x. \end{aligned} \tag{4}$$

The initial conditions are $u'_{\rm d}(x, 0) = 0$ and $\dot{u}'_{\rm d}(x, 0) = 0$.

Equation (3) with boundary conditions (4) can be solved by expanding the dynamic reaction into a series in modes of natural oscillations. In this case,

$$u_{\rm d}(x, t) = \sum_{i=1}^{\infty} q_i(t) X_i(x).$$

The eigenfunctions $X_i(x)$ and the frequencies of natural oscillations p_i are determined by numerical or analytical methods. If the mass of the cable is substantially smaller than that of any of the end bodies ($mL \ll M_i$), then simple approximate models are suitable for the natural frequencies and modes. The first tone of the oscillations is described by the "two masses on a spring" model, while the subsequent tones are described by the "rod with closed ends" model. To obtain the equations of motion for the generalized coordinates q_i, we use the Lagrange equations of the second kind in the form

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_i}\right) + \frac{\partial U}{\partial q_i} = F_i(t).$$

Here

$$K = \frac{1}{2} m \int_L \left(\sum_i \dot{q}_i(t) X_i(x) \right)^2 dx + \frac{1}{2} M_1 \left(\sum_i \dot{q}_i(t) X_i(x_1) \right)^2 + \frac{1}{2} M_2 \left(\sum_i \dot{q}_i(t) X_i(x_2) \right)^2.$$

Then

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{q}_i} = m \int_L \ddot{q}_i(t) X_i^2(x)\, dx + M_1 \ddot{q}_i(t) X_i^2(x_1) + M_2 \ddot{q}_i(t) X_i^2(x_2) = M_{{\rm red}i}\, \ddot{q}_i(t),$$

where $M_{{\rm red}i}$ is the reduced mass of the OCS with oscillations in the ith tone;

$$U = \frac{1}{2} {\rm EF} \int_L \left(\sum_i q_i(t) X'_i(x) \right)^2 dx; \quad \frac{\partial U}{\partial q_i} = {\rm EF} \int_L q_i(t) X_i'^2(x)\, dx = S_{{\rm red}i}\, q_i(t),$$

where $S_{\text{red}i}$ is the reduced coefficient of stiffness with oscillations in the ith tone.

The equations for the generalized coordinates q_i take the form

$$M_{\text{red}i}\,\ddot{q}_i + S_{\text{red}i}\,q_i = F_i\,(t)\ . \tag{5}$$

The expression for the generalized force can be obtained by reducing the inertial forces due to the temperature deformation to the displacements in the ith tone:

$$F_i\,(t) = -\,\alpha\left[m\int_L \ddot{T}xX_i\,(x)\,dx + M_1\ddot{T}x_1X_i\,(x_1) + M_2\ddot{T}x_2X_i\,(x_2)\right].$$

If the dissipative properties of the cable are taken into account, then Eq. (4) is supplemented with a term describing viscous resistance which is equivalent in an energy sense to the inelastic resistance of the cable. The tension of the viscoelastic cable is determined by the expression

$$N = \text{EF}\,(\varepsilon + \beta\dot{\varepsilon})\ . \tag{6}$$

The coefficient of equivalent viscous resistance β can be determined experimentally for the prescribed frequency and the range of deformation amplitudes. Equation (5) with account for Eq. (6) has the form

$$M_{\text{red}i}\,\ddot{q}_i + R_{\text{red}i}\,\dot{q}_i + S_{\text{red}i}\,q_i = F_i\,(t)\ . \tag{7}$$

In this case, the reduced coefficient of viscous resistance of the ith harmonic $R_{\text{red}i}$ is related to the reduced coefficient of stiffness: $R_{\text{red}i} = \beta S_{\text{red}i}$. The numerical solution of Eqs. (5) or (7) with known right-hand sides of the equations ($F_i(t)$) is described in the literature [1] and involves no difficulties. The total displacement of the cable and of the end bodies is determined by the sum (2).

Thus, in the dynamic model proposed, the load is caused by the inertial forces from the prescribed temperature displacements. The variability of the gravitational field and the presence of transfer inertial forces associated with the orbital motion of the OCS are here disregarded.

The low (near-earth) orbit contains portions, one of which is illuminated by the sun, while the other is in shadow. Between them, one separates two half-shadow portions. We assume that the temperature of the cable is the same at all its points along the length and over the cross section. The change in the temperature with time is determined by the equation [2]

$$cm\,\frac{dT}{dt} = A_{\text{s}}\,(Q_1 + Q_2) + \varepsilon_* Q_3 - \varepsilon_*\sigma T^4\pi d\ , \tag{8}$$

where Q_1 is the flux of direct solar radiation, incident on the cable, per unit length of the cable, Q_2 is the flux of solar radiation, incident on the cable, that is reflected from the earth's surface and reduced to unit length of the cable, and Q_3 is the flux of intrinsic radiation of the earth, incident on the cable, that is reduced to unit length of the cable.

The procedure for calculating the quantities Q_1, Q_2, and Q_3 is given in [2]. In the present work, we consider the OCS being in circular orbit in a gravitational-balanced, i.e., vertical, state. The thermal regime of the cable is determined as the regime of a vertical circular cylinder. To carry out the calculations, one must know the cable characteristics c, m, A_{s}, and ε_*. Using the known parameters of the orbit and the prescribed time, it is possible to determine the position of the OCS in orbit (the illuminated portion, shadow, and half-shadow) and also the angle between the cable and the direction to the sun [3].

As an initial condition for Eq. (8), we prescribe the equilibrium temperature ($dT/dt = 0$). Figure 2 presents the result of calculation of the cable temperature $T(t)$ for a circular orbit of height 400 km and in-

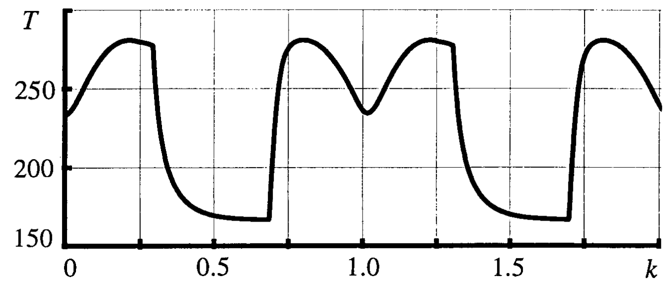

Fig. 2. Change in the cable temperature in two circuits of the orbit. T, K; k is the number of circuits of the orbit.

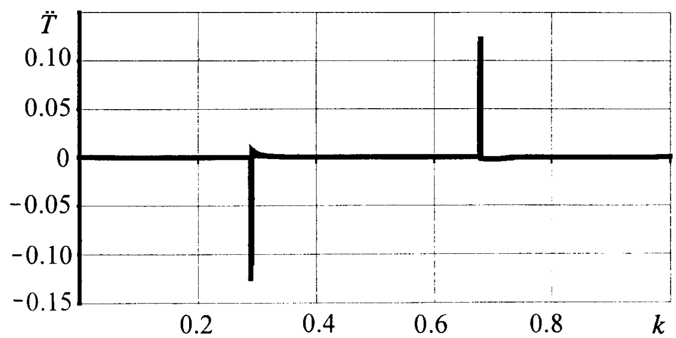

Fig. 3. Second derivative of the cable temperature with respect to time. $\ddot{T}$, K/sec^2; k is the number of circuits of the orbit.

clination 51.6°. At the instant of time $t = 0$, the OCS is located at an ascending node; its longitudinal axis is directed to the sun, which is situated in the orbit plane. We take the following mechanical and thermophysical characteristics of the cable: $c = 1000$ J/(kg·K), $m = 0.005$ kg/m, $A_s = 0.6$, and $\varepsilon = 0.9$. The temperature curve in the orbit circuit experiences two sharp fractures which correspond to the intersections of the terminator line. The duration of the half-shadow for this orbit is ~10 sec. At this instant, the highest rate of change of the cable temperature is 0.65 K/sec. On the shadow portion, whose duration is about 40% of the circuit, the cable temperature decreases smoothly to ~170 K and reaches virtually a stationary value. On the illuminated portion, the temperature reaches 280 K. Here there are two maxima that correspond to the positions of the cable system along the normal to the direction to the sun. The local minimum between them is observed when the sun is at the zenith of the OCS.

The form of the function $\ddot{T}(t)$ for the orbit circuit is shown in Fig. 3. The graph depicts two pulses with different signs that correspond to the intersections of the terminator line. For the remaining period of time, $\ddot{T}(t) \approx 0$ on the scale of this graph. Consequently, the thermoelastic disturbance of the OCS is a sequence of pulses. The behavior of the function $T(t)$ varies from circuit to circuit due to the precession of the orbit plane and the change of the angular coordinates of the sun in inertial space. For periods of time shorter than one day these factors are small [3] and the function $T(t)$ can be considered to be periodic. Here no consideration is given to other factors (for example, the change in the optical properties of the underlying surface of the earth) capable of affecting the thermal state of the cable in orbital motion. Under this assumption, the thermoelastic action on the OCS is periodic and its frequency properties are revealed in expansion of the function $\ddot{T}(t)$ into a Fourier series:

$$\ddot{T}(t) = H_0 + \sum_k H_k \cos(k\omega t + \varphi_k), \tag{9}$$

where H_k and φ_k are the amplitude and phase spectra.

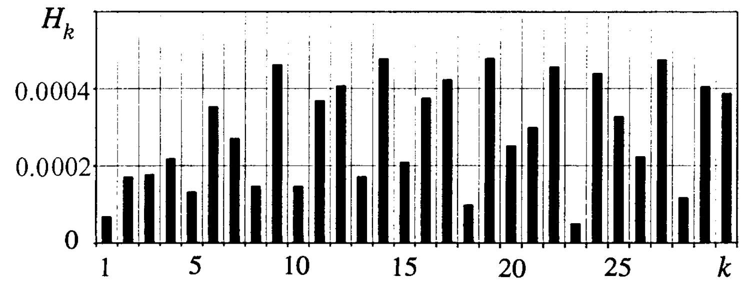

Fig. 4. First thirty harmonics of the amplitude spectrum H_k. H_k, K/sec^2; k is the number of the harmonic.

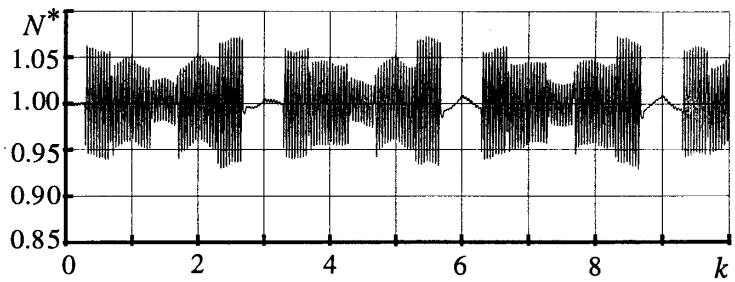

Fig. 5. Change in the tensile force of the cable in ten circuits of the orbit under the temperature action on the OCS (p_1 = 23.5 Ω) without account for the dissipative properties of the cable.

Figure 4 illustrates the amplitudes of the first thirty harmonics of the amplitude spectrum of (9). It can be assumed that the resonant excitation of the longitudinal oscillations of the OCS is possible when any natural frequency coincides with one of the harmonics of (9).

For numerical simulation of the longitudinal oscillations we took the following characteristics of the OCS: $M_1 = 6 \cdot 10^3$ kg, $M_2 = 10^5$ kg, L = 20 km, $m = 5 \cdot 10^{-3}$ kg, and EF $= 8 \cdot 10^4$ N. Such parameters correspond to the circular natural frequencies $p_1 = 0.264 \cdot 10^{-1}$ sec^{-1}, p_2 = 0.752 sec^{-1}, p_3 = 1.50 sec^{-1}, and p_4 = 2.25 sec^{-1}. The circular frequency of the orbiting Ω for the orbit of radius $6.771 \cdot 10^6$ m is $0.113 \cdot 10^{-2}$ sec^{-1}. Consequently, the relation $p_1/\Omega = 23.5$ is obtained. Figure 5 presents the result of calculating the dimensionless tensile force of the cable $N^* = (N_{st} + N_d)/N_{st}$ for ten circuits of the orbit. The dynamic-response factor of the tensile force $\xi = \frac{N_d}{N_{st}} \cdot 100\%$ reaches 7%. The above case of the relation between the frequencies p_1 and Ω is favorable from the viewpoint of the OCS dynamics. Indeed, when the relation of the indicated frequencies has the form $1/(n + 1/2)$, the longitudinal oscillations of the OCS, excited on two successive circuits of the orbit, have the opposite phases and quench each other.

By varying the stiffness of the connecting cable or the masses of the end bodies, we obtain the case of the multiple relation between p_1 and Ω. The result for the dimensionless tensile force with $p_1/\Omega = 24$ is shown in Fig. 6a. Here one can observe a continuous rise in the oscillation amplitude of the tensile force, and in eight circuits of the orbit the dynamic component reaches the value of the static component, i.e., the complete attenuation of the cable occurs.

The dissipative properties of the cable can be characterized by a relative hysteresis Ψ which is related to the coefficient of equivalent viscous resistance β at the prescribed oscillation frequency p: $\Psi = 2\pi p\beta$. In Fig. 6b, we present a result that corresponds to the case of Fig. 6a but for Ψ = 0.09. Here the oscillations between two adjacent pulses have time to damp out substantially. The dynamic-response factor of the tensile

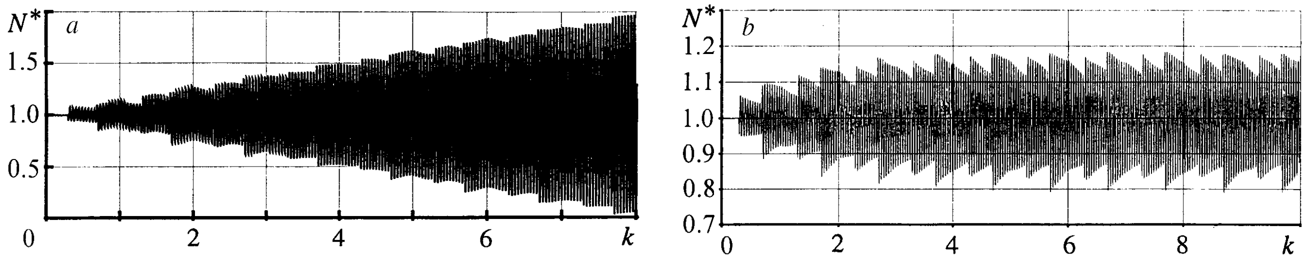

Fig. 6. Tension of the cable at resonance (p_1 = 24 Ω): (a) without account for the dissipative properties of the cable and (b) with account for the dissipative properties of the cable (Ψ = 0.09).

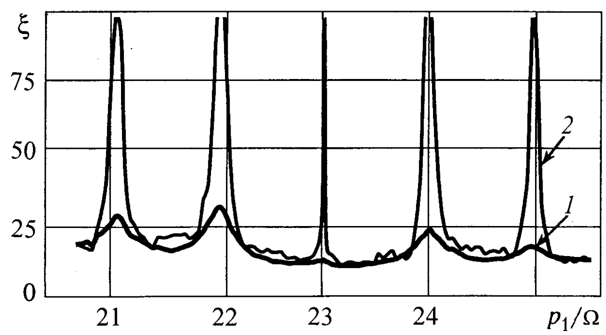

Fig. 7. Frequency characteristic of the dynamic-response factor ξ (%) of the tensile force of the cable: 1) with account for the dissipative properties of the cable (Ψ = 0.09) and 2) without account for the dissipative properties of the cable (Ψ = 0); p_1 is the circular frequency of the fundamental tone of the OCS longitudinal oscillations.

force amounts to 20%. The value of Ψ = 0.09 should be considered to be extremely high for the cables and ropes assumed to be used as part of the OCS. Figure 7 illustrates the frequency characteristic (FCh) for the dynamic-response factor of the tensile force of the cable. In the frequency range, the frequency characteristic covers five harmonics of the thermoelastic disturbance (from the 21st to the 25th) and has five resonance peaks, respectively. The graph depicted in Fig. 7 shows two curves that correspond to the cases Ψ = 0 and Ψ = 0.09. The time in which the complete attenuation of the cable at resonance in the conservative model of the OCS is attained can approximately be estimated analytically, considering the resonance in a system with one degree of freedom under the action of the harmonic disturbing force:

$$\ddot{x} + p^2 x = f \sin \omega t \,.$$

The displacements $x(t)$ at resonance are determined by the relation

$$x(t) = A(t) \cos(\omega t - \varphi) \,,$$

where

$$A(t) = -\frac{f}{2\omega^2}\sqrt{(\omega t)^2 + 1} \,.$$

In this case, the time in which the oscillation amplitude attains the prescribed quantity B is determined in the following manner:

$$t = \left(\frac{4B^2\omega^2}{f^2} - \frac{1}{\omega^2} \right)^{1/2} . \quad (10)$$

Here, as the one-mass system we consider the OCS in the case of longitudinal oscillations in the fundamental tone, B is the elongation of the cable due to the static tensile force (for the prescribed parameters of the OCS and the orbit N_{st} = 437 N), and f is the amplitude of the 24th harmonic of the thermoelastic disturbance divided by the reduced mass of the fundamental tone of the OCS longitudinal oscillations. The analytical estimation according to formula (10) gives a time of 8.96 circuits of the orbit for the complete attenuation of the cable. This result agrees well with that given in Fig. 6a, where a more complex dynamic model of an OCS is subjected to periodic nonharmonic action.

The OCS possesses the infinite spectrum of natural frequencies of longitudinal motion. The spectrum of thermoelastic action is also infinite. In view of this, it is necessary to answer the question as to how many tones of oscillations must be taken into account in the dynamic model of longitudinal motion of an OCS. If the frequency of the first tone of the OCS considered lies in the region of 23–24 harmonics of thermoelastic action, then the second tone lies already in the region of 670–680 harmonics. The amplitudes of the high-frequency harmonics of the disturbing action decrease noticeably. Moreover, when account is taken of the high-frequency components, the external action can no longer be considered to be periodic. The change in the optical properties of the earth's surface on different circuits of the orbit and other disregarded factors will be substantial here. Consequently, the resonant excitation of oscillation tones above the first one is impossible.

Thus, the analysis made and the numerical modeling have shown the possibility of the longitudinal oscillations of the orbital cable system being resonant-excitated under the thermal action on the cable in orbital motion. To eliminate dangerous dynamic phenomena, the fundamental tone of the longitudinal oscillations of the OCS must not be a multiple of the orbiting period.

NOTATION

α, coefficient of linear expansion; β, coefficient of viscous resistance of the cable; ε, deformation; ε_*, emissivity; φ, phase of oscillations; σ, Stefan–Boltzmann constant; Ω, circular frequency of orbiting; ω, circular frequency; ξ, dynamic-response factor of tension of the cable; Ψ, coefficient of energy absorption in the case of oscillations (relative hysteresis); A, amplitude of forced oscillations; a, velocity of sound in the cable; A_s, coefficient of radiation absorption; B, elongation of the cable due to the static tensile force; C, specific heat of the cable; d, cable diameter; EF, stiffness of the cable in tension; F, generalized force; f, amplitude of the disturbing force; H, amplitude spectrum; K, kinetic energy; k, wave number, number of circuits of the orbit, and harmonic number in the spectrum; L, length of the cable system; M, mass; m, mass per unit length; N, tension of the cable; N^*, dimensionless tension; n, integer; p, circular frequency of natural oscillations; Q, heat flux; q, generalized coordinate; R, coefficient of viscous resistance; S, stiffness coefficient; T, temperature; ΔT, temperature change; t, time; U, potential energy; u, longitudinal displacement; X, mode of natural oscillations; x, coordinate. Subscripts: i, number of the end mass; j, number of the natural-oscillation tone; red, reduced; th, thermal; d, dynamic; st, static.

REFERENCES

1. S. P. Timoshenko, D. H. Young, and W. Weaver, in: *Vibration Problems in Engineering* [Russian translation], Moscow (1985), pp. 93–95.
2. V. V. Malozemov, in: *Thermal Mode of Spacecraft* [in Russian], Moscow (1980), pp. 46–49.
3. G. M. Chernyavskii and V. A. Bartenev, in: *Orbits of Communications Satellites* [in Russian], Moscow (1978), pp. 49–51.